Re-examining the role of financial constraints in business cycles: is something wrong with the credit multiplier?*

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September 2010

Abstract

A large theoretical literature suggests that financial frictions provide a mechanism which amplifies and propagates macroeconomic shocks. However, quantitative papers that embed this mechanism, referred to as the credit multiplier, into standard DSGE models conclude that although credit constraints delay the velocity at which productivity shocks propagate into the economy, they have no significant amplification effects, with the exception of special cases. Motivated by these results, in this paper we re-examine the quantitative role of financial frictions in business cycles to address the following question: is there something wrong with the credit multiplier? Our answer is no. In coming to this answer, we work with a model with reproducible capital and collateral constraints within two setups, a general and a partial equilibrium. Our results from the first model in terms of propagation and amplification do not differ from previous papers. However, our main finding is that it is not the credit multiplier what fails in this type of models, but rather their ability to produce sufficient variability in prices. In particular, in a model with reproducible capital, general equilibrium dynamics counteract the logic of price fluctuations described by theoretical models, thus preventing the credit multiplier from being triggered. The partial equilibrium setup allows us to confirm our previous claim: absent the general equilibrium effects, the credit multiplier is indeed an effective amplifying mechanism of shocks into the economy.

*We are very grateful to Christian Hellwig and Lee Ohanian for their valuable comments and support. We also thank Andrew Atkeson, Ariel Burstein, Mark Wright, and participants of the Macro Economics Proseminar at UCLA, the 2010 Society for Economic Dynamics Meeting and the 2010 European Economic Association Congress for helpful comments.
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1 Introduction

"Business cycles appear to be large, persistent and asymmetric relative to the shocks hitting the economy".[1] One of the explanations that have been offered as to what lies behind this observation, states that there is some underlying mechanism in the economy that transforms small, temporary shocks into large and persistent aggregate output fluctuations. In this line, many theoretical papers suggest that financial frictions provide this mechanism, and thus play a key role in amplifying and propagating macroeconomic shocks (see Bernanke and Gertler (1989) and Kiyotaki and Moore (1997)). These papers model financial frictions as endogenous borrowing constraints that depend on the degree of agents' solvency and limit the extent to which productive projects can be externally financed. In particular, Kiyotaki and Moore (1997), henceforth KM, consider an economy where assets serve both as factors of production and collateral for loans. The fact that assets have two roles is key, since it determines that not only will agents' borrowing constraint limits depend on the value of their assets, but also, the price of these assets will be affected by the size of the credit limits. The dynamic interaction between asset prices and credit limits that is triggered when the economy is hit by a shock is thus capable of generating large and persistent fluctuations in output and asset prices. This mechanism is usually referred to as the credit multiplier. Despite theoretical consensus on the key role of financial frictions in amplifying and propagating macroeconomic shocks, quantitative papers that embed the credit multiplier into fully fledged DSGE models conclude that although credit constraints delay the velocity at which productivity shocks propagate into the economy, they have no significant amplification effects, with the exception of special cases.[2]

Motivated by these results, in this paper we re-examine the quantitative role of financial frictions in business cycles to address the following question: is there something wrong with the credit multiplier? Our answer is no. In arriving to this conclusion, we work with a model with reproducible capital and collateral constraints within two setups, a general and a partial equilibrium. Our results from the first model in terms of propagation and amplification do not differ from previous papers. However, we find that it is not the credit multiplier what fails in this type of models, but rather their ability to produce sufficient variability in prices. In a model with reproducible capital, this happens because the general equilibrium dynamics dampen the response of asset prices to a productivity shock, thus reducing their effect on borrowing constraints and preventing the credit multiplier from being triggered. Our partial equilibrium setup allows us to confirm the previous claim: when isolated from general equilibrium effects, the credit multiplier is indeed effective in amplifying shocks, generating large investment, output and

[1] This is how Kocherlakota (2000) describes output fluctuations over the years. See Kocherlakota (2000), p. 2.

[2] See, for instance, Kocherlakota (2000) and Cordoba and Ripoll (2004a) for models with a fixed factor in which amplification only arises under specific combinations of parameters. Carlstrom and Fuerst (2001), Chen (2001), Krishnamurthy (2003), Cordoba and Ripoll (2004b), Aoki, Benigno and Kiyotaki (2009) and Liu, Wang and Zha (2009) are other examples of models in which amplification may occur; however, their setups constitute major departures from the standard RBC model.

price fluctuations.

We begin our analysis by introducing collateral constraints in the production of capital goods in an otherwise standard RBC model. The fact that capital is reproducible and that it is the only asset that can be used as collateral in the economy is one of the main distinctions of the model with respect to other papers that have tested the propagation and amplification of shocks in models with collateral constraints (see, for example, Kocherlakota (2000), Cordoba and Ripol (2004a) and Liu, Wang and Zha (2009)). In our setup, collateralized debt arises from a limited commitment problem on the borrower's side and collateral is given by the value of total production of the constrained agent at the end of each period. A key feature then is that collateral is not fixed and thus its value may vary not only because of price changes but also due to quantity changes.

We find that a model with these features is capable of delivering more than 25% more propagation relative to its frictionless counterpart; however, in terms of amplification, not only does the model not display amplification of the magnitude predicted by the theory, but it also performs worse than a model with the same characteristics but without frictions (roughly 20% less amplification). The main reason underlying the last result is that when we allow capital to be reproducible, the general equilibrium effects that arise counteract the logic of asset price fluctuations described in theoretical models. The generated volatility is not only not enough to generate the necessary increase in the price of capital so as to obtain quantitatively significant amplification but is also substantially smaller than that observed in the data (0.56% versus 10.35%, respectively).[3] The intuition behind this is the following: when the economy is hit by a positive productivity shock, the relative scarcity of capital generated is partially offset by the increase in investment that takes place immediately after the shock, which in turn dampens the variability of asset prices. One could argue that the small increase in prices could potentially be offset by the increase in investment, however, in our model this increase is much lower than the one that would be needed to offset the small price effect on credit limits. Thus, the countervailing effect observed prevents the credit multiplier mechanism from being triggered and hence is key to understand the lack of amplification under a general equilibrium setup.[4]

This motivates our second exercise, a partial equilibrium analysis, in which we isolate the credit multiplier from general equilibrium dynamics to test whether, when being fed with empirically plausible changes in asset prices, it delivers a higher degree of output amplification. Specifically, we abstract from household behavior and focus on the lending relationship and investment decisions of entrepreneurs, taking the volatility of the price of capital as given. As mentioned before, the results from this exercise confirm our claim that, absent the general equilibrium effects, the credit multiplier is an effective am-

[3] The price of capital is approximated by the S&P 500 market price deflated by the price of non-durable goods and services. All volatilities are computed as the standard deviation of the HP-filtered data from 1952:Q1 to 2009:Q3.

[4] Our results are robust when changing relevant parameter values, adding capital adjustment costs, and adding land as an additional input of production and collateralizable asset.

plifying mechanism of shocks into the economy. Furthermore, when calibrated to match realistic price movements, our partial equilibrium model generates investment volatility of the same order of magnitude as those obtained in the general equilibrium model and in the RBC (4.85%, 4.06% and 7.27%, respectively).

Our model is closely related to that of Carlstrom and Fuerst (1997), henceforth CF, who develop a computable general equilibrium model where physical capital producers face endogenous agency costs that limit their access to credit. Their main findings suggest that although agency costs cause a delay in the change of aggregate investment, enhancing the persistence of productivity shocks which in turn help replicate the hump-shaped response of output found in the data, they fail to produce an amplified response of output to shocks. We depart from CF's framework in that borrowing constraints arise from limited enforceability of contracts, instead of from endogenous agency costs. In this way, we are able to test the quantitative significance of the credit multiplier as described by KM and revise the role that asset prices play in generating persistence and amplification within a setup that, if financial frictions were removed, collapses to a standard RBC model. Our paper contributes to the literature on financial frictions by showing that the lack of amplification in these models should not be attributed to the failure of the credit multiplier mechanism but rather to the general equilibrium dynamics that dampen the variability of asset prices and do not allow the credit multiplier to come to be.

The rest of the paper is organized as follows. Section 2 presents the benchmark general equilibrium model. Section 3 describes its calibration and main results. Section 4 introduces the partial equilibrium version of our economy model and comments on its results. Finally, Section 5 concludes.

2 The benchmark model

2.1 General features of the economy

Consider an economy populated by a continuum of infinitely lived agents of two types, households and entrepreneurs. Both types of agents have unit mass. The entrepreneurs produce capital goods from consumption goods using a linear technology, for which they need external financing as well as their own income. The economy is also populated with a firm producing a single consumption good, and a risk neutral capital mutual fund (CMF) that acts as intermediary between the households' resources and entrepreneurials' financing needs.

A productivity shock to the consumption goods sector is realized at the beginning of the period. The sequence of events within a given time period following the realization of this shock is the following:

1. The consumption goods producing firm hires labor and rents capital from households and entrepreneurs and production takes place.

2. Households decide how much to consume and invest. For each unit of capital that a household wishes to purchase, she gives q_t units of consumption goods to the CMF, where q_t is the price of capital.

3. The CMF uses the resources obtained from households to provide loans to the entrepreneurs.

4. Entrepreneurs invest the resources borrowed, together with their own net worth, into their capital production technology.

5. Finally, entrepreneurs repay their loans and make consumption and investment decisions.

An important feature of this economy is that if a household wishes to purchase capital she will do so by lending funds to the entrepreneurs through the CMF. The CMF will then lend the resources to the entrepreneur and a loan contract will be established. In this contract debt is collateralized, thus making credit limits endogenous. This arises because entrepreneurs cannot commit to repay their loans, and therefore must provide a collateral as a way of securing their debts. The financial contract will be explained in detail when we talk about the problem of the entrepreneur. Having described briefly the general features of the economy, we go on now to describe the problem of each agent.

2.2 Households

The representative household's problem is given by:

$$\max_{\{c_t^H,\, k_{t+1}^H,\, l_t^H\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} \beta^t U(c_t^H, 1 - l_t^H)$$

$$s.t. \qquad c_t^H + q_t(k_{t+1}^H - (1-\delta)k_t^H) \leq w_t^H l_t^H + r_t k_t^H$$

where E_t denotes the expectational operator conditional on time t information, $\beta \in (0,1)$ is the time discount factor, c_t^H, k_t^H and l_t^H are household's consumption, stock of capital and labor supply, respectively, all in period t. Leisure endowment is normalized to one. Every period, the household sells her labor and rents last period's accumulated capital to the consumption good producing firm at a wage rate w_t^H and rental rate r_t, respectively. They also purchase consumption goods at a price of 1, and capital goods, with the assistance of the CMF, at price q_t.

Household's choices are summarized in the Euler equation:

$$q_t = \beta E_t\left[\frac{U_c(c_{t+1}^H, 1 - l_{t+1}^H)}{U_c(c_t^H, 1 - l_t^H)}(r_{t+1} + (1-\delta)q_{t+1})\right] \tag{1}$$

and the labor leisure condition:

$$U_l(c_t^H, 1 - l_t^H) = U_c(c_t^H, 1 - l_t^H)w_t^H \tag{2}$$

where δ is the rate of depreciation on capital.

2.3 Consumption good producing firm

Consumption goods are produced using a standard constant returns to scale production function. The firm's problem is given by:

$$\max_{\{k_t^d,\ l_t^{Hd},\ l_t^{Ed}\}} F(k_t, l_t^H, l_t^E) - r_t k_t - w_t^H l_t^H - w_t^E l_t^E$$

$$\text{where} \qquad F(k_t, l_t^H, l_t^E) = \theta_t (k_t)^{\alpha_k} (l_t^H)^{\alpha_h} (l_t^E)^{\alpha_E} \tag{3}$$

$F(k_t, l_t^H, l_t^E)$ denotes aggregate output, k_t, l_t^H and l_t^E are the firm's demand for capital, household and entrepreneurial labor, respectively. α_k, α_h and α_E are their respective shares in output and w_t^E is the wage rate for entrepreneurial labor. Finally, θ_t is the productivity shock. Competition in the factor markets implies that wages and the rental rate are equal to their marginal products:

$$r_t = \alpha_k F_t / k_t \tag{4}$$

$$w_t^E = \alpha_E F_t / l_t^E \tag{5}$$

$$w_t^H = \alpha_H F_t / l_t^H \tag{6}$$

2.4 Entrepreneurs

It is useful to define the representative entrepreneur's problem as consisting of 5 steps:

1. The entrepreneur rents his capital and inelastically supplies his labor to the consumption goods producing firm. After the production of consumption goods takes place, he sells his undepreciated capital to the CMF for consumption goods to build up net worth (expressed in consumption units):

$$n_t = w_t^E + k_t^E [q_t(1-\delta) + r_t] \tag{7}$$

where n_t is net worth and k_t^E denotes capital holdings of the entrepreneur at the beginning of the period.

2. He borrows resources from the households, through the CMF, using his net worth as the basis of the loan contract:

$$i_t - n_t$$

where i_t is investment at time t.

Two key assumptions about the entrepreneur are implicit in the contract. First, the entrepreneur's technology is idiosyncratic in the sense that, once his production has started, only he has the skill necessary to finish it and second, the entrepreneur cannot precommit to repay his loan.[5] This makes

[5] To make his commitment problem relevant, we assume that the entrepreneur will always want to invest more than his net worth. This implies assuming that net worth is sufficiently small which is achieved by making the entrepreneur more impatient than the household.

the creditors want to secure themselves by not allowing the value of debts to exceed the value of the entrepreneur's collateral, which in turn makes borrowing constraints endogenous in this contract. More specifically, through the contract, the entrepreneur agrees to borrow $i_t - n_t$ and to repay d_t to the lender at the end of the period, after all production has taken place. He will choose to default if the value of his production net of repayment is lower than the value he obtains when defaulting, given by a fraction χ of the value of his production.[6] That is, the entrepreneur will choose to maximize the following expression:

$$\max\{\underbrace{q_t z i_t - d_t}_{\text{if no default}};\ \underbrace{\chi q_t z i_t}_{\text{if default}}\}$$

where zi_t is the entrepreneur's capital goods production technology with z representing his exogenous productivity. Thus, for the contract to be self-enforcing, d_t must be set such that the entrepreneur will always choose not to default in equilibrium. This condition is given by:

$$d_t \leq (1-\chi)q_t z i_t$$

which ensures that:

$$\max\{q_t z i_t - d_t; \chi q_t z i_t\} = q_t z i_t - d_t$$

Hence, the optimal contract is given by the solution to:

$$\max_{\{i_t\}} q_t z i_t - d_t$$

$$\begin{array}{rrcl} s.t & i_t - n_t & = & d_t \\ & (1-\chi)q_t z i_t & \geq & d_t \\ & q_t z i_t - d_t & \geq & n_t \end{array}$$

where the first constraint represents the break-even condition for the lender, and the second and third represent the borrowing constraint and the participation constraint for the entrepreneur, respectively. Two key things are worth noting here. First, the participation constraint will never bind, which will ensure that the return to internal funds is greater than the return to external funds (which is one, given the intra-period nature of the loans), thus providing incentives for the entrepreneur to accumulate as much net worth as possible and invest it all in his project.[7] Second, the borrowing constraint will always bind. This is due to the linear nature of the capital goods production technology as well as to the return to internal funds being greater than one, both of which make the entrepreneur want to invest as much as possible to maximize his income.

[6] Note that there is no penalty from defaulting, if the entrepreneur defaults one period, he is not banned from participating in the loan market in the next. Thus, the contract is based solely on entrepreneurial net worth and not on past outcomes.

[7] The model will be calibrated so as to ensure this is true.

Finally, from the break-even condition and the borrowing constraint, we can solve for the amount of loans and investment:

$$d_t = \frac{(1-\chi)q_t z}{1-(1-\chi)q_t z} n_t$$

and

$$i_t = \frac{1}{1-(1-\chi)q_t z} n_t \tag{8}$$

From these equations we can see that investment, and thus capital production, as well the total amount of loans, are increasing functions of the entrepreneur's net worth n_t and the price of capital q_t. This is what lies at the heart of the credit multiplier described in theoretical papers on collateral constraints, that is, the mechanisms capable of amplifying and propagating macroeconomic shocks.

3. He invests i_t in his capital goods production technology.

4. The entrepreneur repays his loan:

$$d_t = (1-\chi)q_t z i_t \tag{9}$$

5. Finally, he consumes and accumulates capital. His maximizing problem at the end of the period is given by:

$$\begin{aligned} &\max_{\{c_t^E,\, k_{t+1}^E\}_{t=0}^{\infty}} \quad E_t \sum_{t=0}^{\infty} (\beta\gamma)^t U(c_t^E) \\ &s.t. \qquad c_t^E + q_t k_{t+1}^E \leq R_t n_t \\ &\text{where} \qquad R_t = \frac{(q_t z i_t - d_t)}{n_t} = \frac{\chi q_t z i_t}{n_t} \end{aligned} \tag{10}$$

where c_t^E is entrepreneurial consumption, k_{t+1}^E is his demand for new capital, R_t is the return to internal funds and $\beta\gamma$ is the time discount factor. We assume here that entrepreneurs discount the future more heavily than households and thus $\gamma \in (0,1)$. As mentioned before, this is to ensure that entrepreneurial consumption is always positive and self-financing never occurs. Without this assumption, and given the high return to internal funds, entrepreneurs would postpone consumption and accumulate as much capital as possible so that they are completely self-financed, thus making the commitment problem and credit constraints irrelevant.

The Euler equation for the entrepreneur is given by:

$$q_t = \beta\gamma E_t\left[\frac{U_c(c_{t+1}^E)}{U_c(c_t^E)}(r_{t+1} + (1-\delta)q_{t+1})\right]R_{t+1} \tag{11}$$

2.5 CMF

The key role of the CMF is to act as an intermediary between households, who want to purchase capital and are ultimately the lenders in this economy, and entrepreneurs, who produce the capital. It thus acts as a cooperative, by which capital can be efficiently purchased. Note that the capital the household buys comes from three different sources: i) undepreciated capital sold by entrepreneurs to the CMF after consumption goods production has taken place, ii) new capital goods the CMF receives as repayment for their loans and iii) new capital that entrepreneurs must sell off to finance their end of period consumption.

2.6 Equilibrium

A competitive equilibrium are sequences of prices $\{r_t, w_t^H, w_t^E, q_t\}_{t=0}^{\infty}$, and allocations for households and entrepreneurs $\{c_t^H, k_{t+1}^H, l_t^H, c_t^E, k_{t+1}^E, l_t^E, i_t, n_t, d_t\}_{t=0}^{\infty}$ such that, given prices and k_0, allocations solve the household's, the consumption goods firm's and the entrepreneur's problems (equations (1) to (11) hold) and markets clear:

$$F_t = c_t^H + c_t^E + i_t \tag{12}$$

$$G_t = zi_t = k_{t+1} - (1-\delta)k_t \tag{13}$$

$$k_t^H + k_t^E = k_t \tag{14}$$

$$l_t^E = 1 \tag{15}$$

3 Calibration and results

In this section we first describe the calibration of the benchmark model, then we compare its impulse response functions to a productivity shock to its frictionless counterpart and comment on the results, and finally, we present a sensitivity analysis.

3.1 Calibration

The model is calibrated on a quarterly basis. Households' discount factor, β, is set to 0.9913, thus implying a real annual interest rate of 4%. Following CF, we assume that $U(c_t^H, 1-l_t^H) = \ln c_t^H + \nu(1-l_t^H)$ where ν is set to 2.91 so as to match a steady state labor participation rate of households of 0.3.

The values of factor shares in the production of final goods, α^K, α^H and α^L, are set to 0.36, 0.6399 and 0.0001, respectively. Notice that entrepreneurial labor share is positive but very small to ensure that net worth is not driven by labor dynamics. The annual depreciation of capital is assumed to be 10%, hence δ is set to 0.025. The value of γ is set to 0.972 so as to match an annual steady state rate of return to internal funds of around 12.31%.[8] Overall, the entrepreneurial discount factor is thus roughly 0.96. We assume that $U(c_t^E) = \frac{(c_t^E)^{1-\phi}}{1-\phi}$ with ϕ equal to 0.01.[9] Finally, we set χ, the fraction of total value of

[8] The average ROE for the main U.S. sectors between 1998 and 2008 equals 12.31%. Data available at http://pages.stern.nyu.edu/~adamodar/.

[9] This assumption differs from CF's linear utility function for the entrepreneur, which implies an IES=∞. Adding a small degree of curvature to the utility function allows us to uniquely pin down entrepreneurial consumption. Notice that the IES in this case is still high (equal to 100) so the results derived from the two cases are quite similar.

production that the entrepreneur gets to keep in case of default, equal to 0.73 to match a leverage ratio of 38%.[10]

We take a standard stance and assume that the aggregate productivity shock, θ_t, follows an AR(1) process: $\theta_t = a + \rho\theta_{t-1} + \varepsilon_t$ with ρ and a equal to 0.95 and 0.05, respectively, so that the unconditional expected value of θ is 1. ε_t is a zero-mean *iid* innovation with a standard deviation of 0.007. We abstract the analysis from idiosyncratic differences and set the productivity parameter in the capital goods technology, z, to 1. Table 1 summarizes the values of the parameters used.

Table 1

Parameter values		
Households' discount factor	β	0.9913
Leisure term	ν	2.91
Capital share	α^K	0.36
Households' labor share	α^H	0.6399
Entrepreneurs' labor share	α^E	0.0001
Depreciation	δ	0.025
Entrepreneurs' additional discount factor	γ	0.9720
Entrepreneurs' coefficient of risk aversion	ϕ	0.01
Fraction of value of production going to entrepreneurs if default	χ	0.73
Productivity of capital production technology	z	1
Autorregresive coeff. productivity shock	ρ	0.95
Constant of autorregresive process	a	0.05
Std. deviation productivity shock	σ	0.007

3.2 Results

3.2.1 Impulse response functions

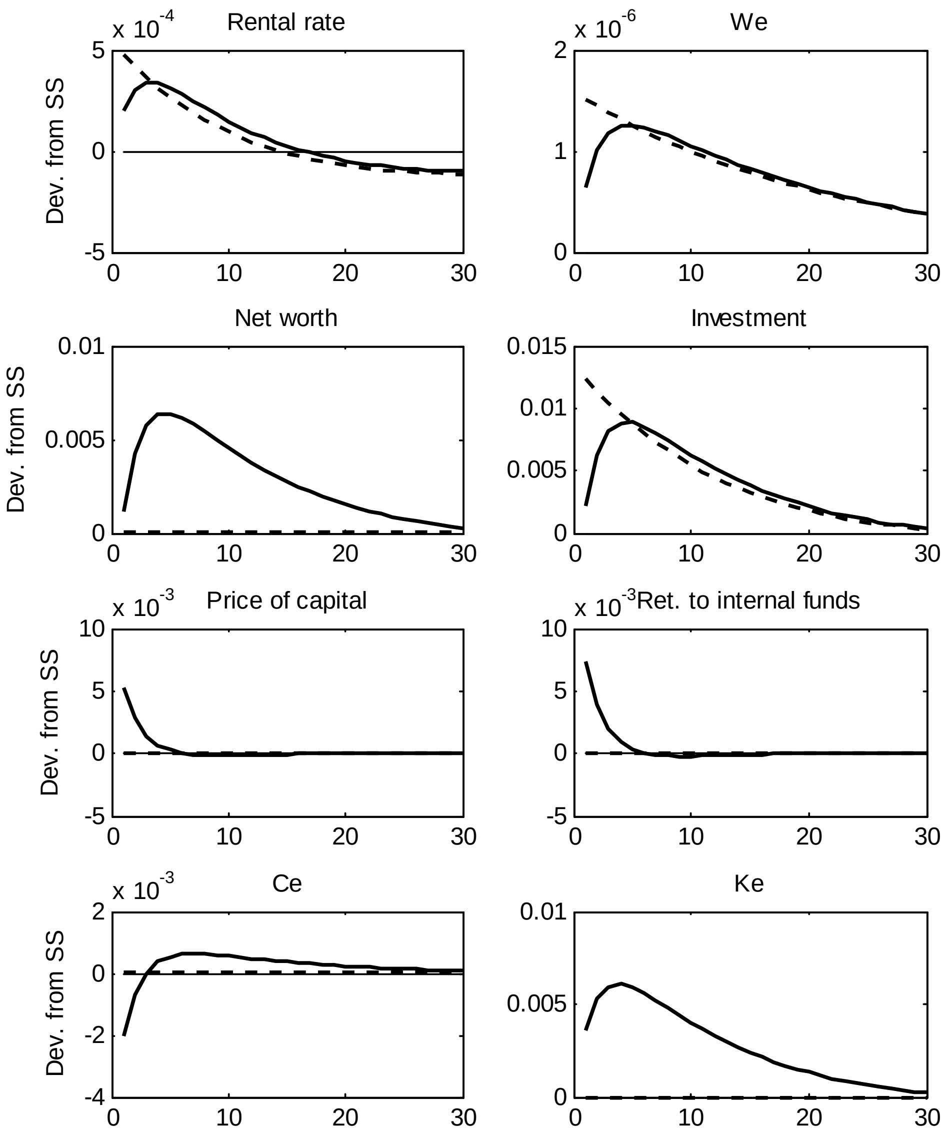
In order to study the role that credit constraints play as an amplification and propagation mechanism of shocks into the economy, we analyze the behavior of our model economy when it is hit by a one standard deviation shock to aggregate productivity. From now on we will refer to our benchmark model as the CC model. Then, we contrast these results with the ones obtained from a model in which collateralized debt is not present; in this case, the economy collapses to a standard RBC model.[11] Figure 1 presents

[10] This is in line with data on average leverage (measured by debt over net worth) over the last 50 years published in the Flow of Funds Accounts of the U.S. by the Federal Reserve.

[11] Notice that when loan contracts are perfectly enforced, the price of capital and hence the return to internal funds, q_t and R_t, are 1 due to the linearity of the technology and the fact that entrepreneurs run their investment projects with zero

the impulse response functions of both models.

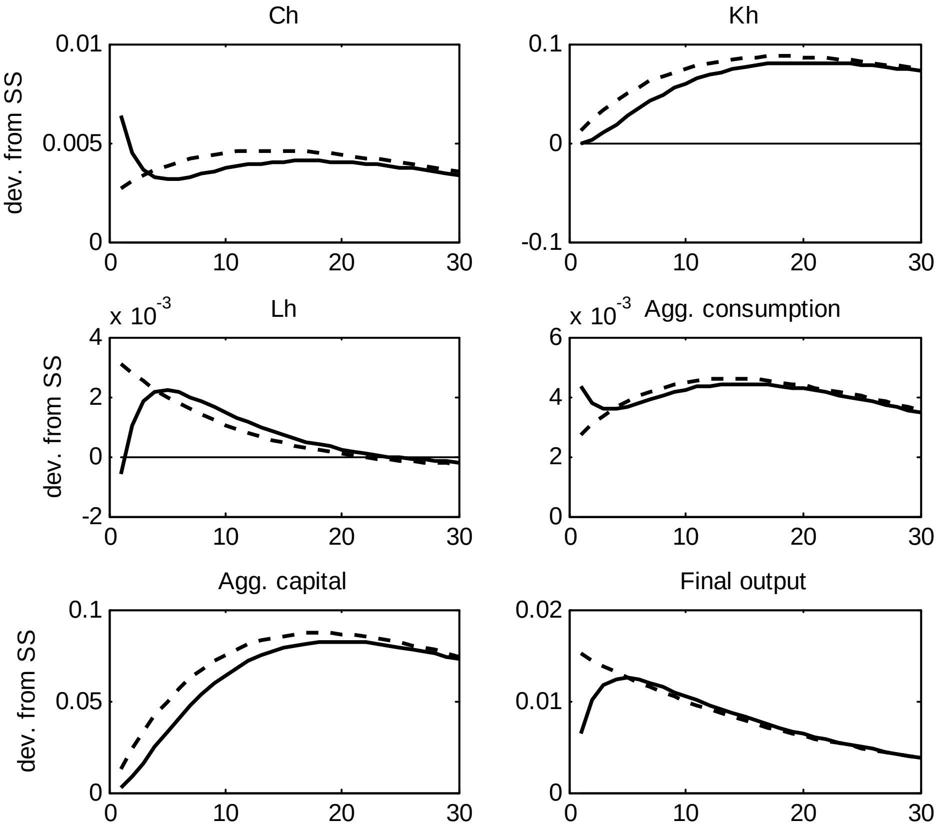
Figure 1a: Impulse response functions



Dynamics of the CC (solid line) and RBC (dotted line) models.

profits. Given that entrepreneurs discount the future more heavily than households, in steady state they do not accumulate any capital and end up consuming all their income. Hence, the economy takes the form of a standard RBC model in which a small fraction of income flows to entrepreneurs.

Figure 1b: Impulse response functions



Dynamics of the CC (solid line) and RBC (dotted line) models.

Figure 1a shows how, at the moment of the shock, the rental rate of capital and wages go up in the CC model, thus increasing the entrepreneur's net worth. Investment also goes up, given that it is proportional to the entrepreneur's net worth. However, since the increase in productivity drives up the demand for capital which in turn increases its price, the increase in investment is greater than the initial increase in net worth (recall investment is also increasing in q_t). Moreover, as returns to internal funds rise, these provide greater incentives for the entrepreneur to increase his capital holdings so as to make his net worth also go up in future periods. Entrepreneurial's higher degree of capital accumulation significantly decreases his consumption at the period of the shock. Notice, in contrast, that in the RBC model the entrepreneur has no incentives to accumulate capital, consequently, the increase in wages is fully transferred to consumption (the magnitude of the increase is so small that can be neglected, though).

As for the household, the RBC model predicts that the increase in her income is translated into more capital accumulation and a higher level of consumption. In contrast, the CC model shows a delayed response in capital accumulation that is compensated by an initial increase in consumption. Under our calibration, as Figure 1b shows, there is a transient income effect that makes her momentarily decrease her supply of labor at the time of the shock.

In the aggregate, the behavior of consumption and capital accumulation mimics that of the household given the fact that the size of the entrepreneur in the economy is relatively small. The most important discrepancy between the CC and the RBC models can be seen in the response of investment, that is directly translated into output. While in the RBC they follow the dynamics of the productivity shock, in the CC model they present a hump-shaped behavior.

3.2.2 Propagation and amplification

Once the transmission mechanism of the CC model has been described we can focus on the results of the model regarding propagation and amplification. Table 2 presents some of the statistics derived from the model that help quantify the persistence and magnitude of the movements observed.

Table 2

Amplification and persistence

	Output		Investment	
Persistence	0.90	(0.71)	0.92	(0.70)
Periods to peak	5	(1)	5	(1)
Max. dev. from ss level (%)	1.25	(1.52)	0.89	(1.24)

Propagation is measured as the positive autocorrelation of a variable that is generated when the economy is subject to a shock. In this sense, the hump-shaped response of some of the variables commented above already predicts an increase in the propagation mechanism displayed by the CC model. We use the first order autocorrelation coefficient of the impulse response function to quantify the degree of propagation of a variable. The first row of Table 2 presents its magnitude for final output and investment in the CC model and contrasts them with their counterparts in the RBC model (in parenthesis). As expected, the CC economy displays more than 25% more propagation than the frictionless economy for both output and investment. It is important to highlight that this increased persistence arises solely due to the different reaction of the variables during the first periods after the shock and not due to the degree of persistence that all variables inherit from the dynamics of the productivity shock itself. Another indicative measure of the delayed response of the business cycle dynamics under financial frictions is given by the number of periods that it takes for the different variables to reach their peak value after a shock occurs. While in the RBC model all variables respond immediately, reaching their peak value in period one (the period of the shock) in the CC model variables reach their maximum value in period five. This is consistent with Cogley and Nason (1995) who show that output in the U.S. peaks four quarters after a shock occurs.

We say that a model displays an amplification mechanism if it succeeds in creating interactions such that the maximum deviation of output from steady state when the mechanism is at work exceeds the maximum deviation reached when the mechanism is not present. As the third row of Table 2 shows, output and investment percentage deviations from their respective steady state in the CC model are smaller relative to those in a model with no frictions (21% and 24% less deviation in each case). This is, our model lacks amplification of productivity shocks.

3.2.3 What lies behind the lack of amplification?

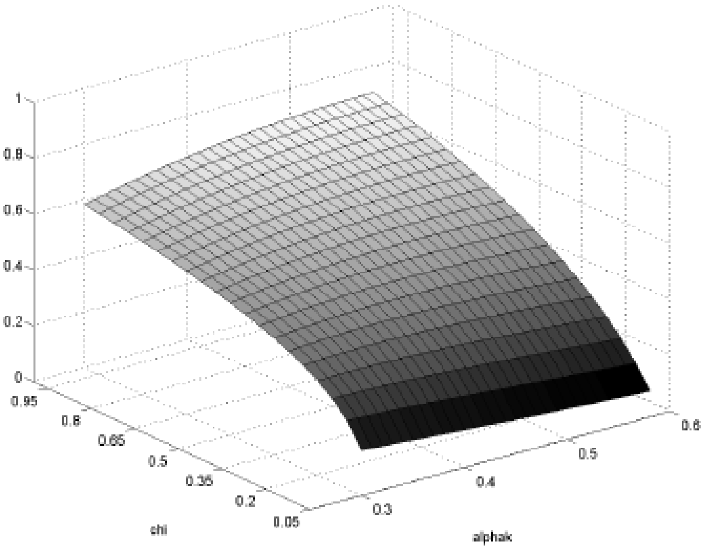
First of all, note that in our model the initial response of output to a productivity shock is smaller than in a model without frictions, and even though output continues to increase in the period following the shock, this is not enough to generate amplification above that obtained by the RBC. This is different from what we observe in a model with a fixed factor, where the initial response of output to a shock is the same in a model with and without frictions. Thus, any further increase in output that occurs in a model with frictions and a fixed factor, will deliver amplification above the one obtained by a frictionless model.

The fact that our model has reproducible capital has a strong implication for prices; this is the main reason behind our lack of amplification. When we allow for capital to be reproducible, the effect that a productivity shock has on its price is greatly reduced relative to when capital is fixed. Thus, its effect on the borrowing constraint is also diminished. One can argue that this could potentially be offset by the increase in investment that takes place when productivity increases, but in our calibration this increase is much lower than the one that would be needed to offset the small price effect on credit limits. We would need then a much higher increase in the price of capital to generate significant capital accumulation by entrepreneurs so as to improve upon a model without frictions.

In order to see what is the maximum increase in price of capital and therefore the maximum amplification that the model is able to deliver, we varied the value of key parameters. Even though we improved relative to our benchmark model, we could never top the model without frictions. In particular, α_k, the share of capital in production, and χ, the parameter that determines the share of the value of total capital production that is used as collateral and hence the amount of loans in the economy, were varied.

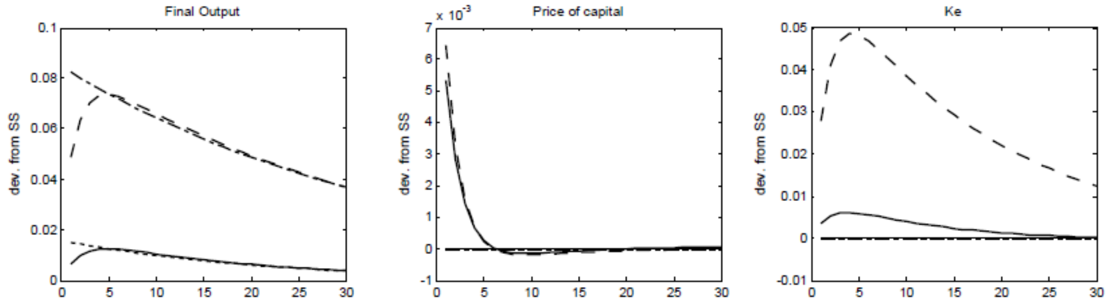
With respect to the price of capital, Figure 2 shows how it changes as these two parameters vary. Observe that the bigger the parameters are, the greater the initial increase in q_t .

Figure 2: Initial increase in the price of capital



With respect to amplification, increasing the capital share at the expense of household's labor share greatly increases amplification. However, this is also true in a model without frictions. Figure 3 illustrates these results. On the other hand, changing χ does not affect the frictionless model's results, but its effect on the constrained model is not as straightforward as the one we get from changing the capital share. In particular, we observe that the larger χ is, the greater the increase in the price of capital, but the smaller the increase in the amount of loans, and leverage, as the borrowing constraint is tightened. However, given that the increase in q_t helps relax the credit constraint, one could argue that if this was large enough it could offset the effect of the higher χ. All in all, this is a quantitative matter and our results suggest that the increase in the price of capital is not large enough to offset the decrease in lending that is brought about by the larger χ, but is sufficient to generate some incentives for the entrepreneurs to accumulate more capital so that the increase in investment and output is almost the same as in our benchmark model. What we have is a change in the composition of investment, in favor of entrepreneurs net worth, but not in its quantity. Also, note that even though the increase in q_t is enough to maintain investment constant, it is not big enough to provide sufficient incentives for total capital accumulation to be above that in the benchmark model, and thus, amplification is not obtained. Figure 4 illustrates these results. Finally, we looked at the effect on amplification from combining changes in factor shares and changes in χ but the results do not change significantly relative to the effects of changing factor shares only, given the small overall effects generated by changing χ. This is shown in Figure 5.

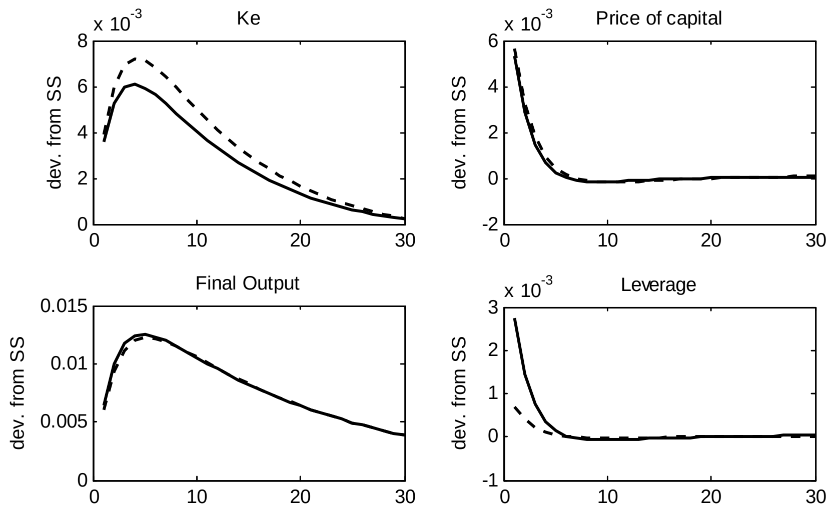
Figure 3: Varying factor shares



The solid and dotted lines depict the CC and RBC models' dynamics with $\alpha = 0.36$, respectively.

The dashed and dash-dotted lines depict the CC and RBC models' dynamics with $\alpha = 0.55$, respectively.

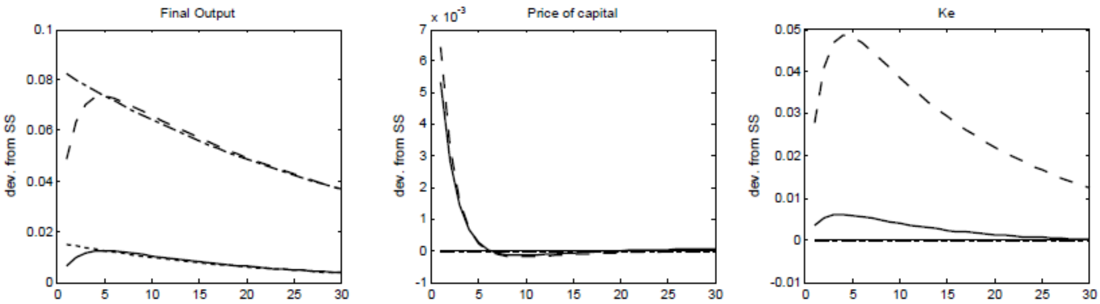
Figure 4: Varying χ



The solid and dotted lines depict the CC model with $\chi = 0.73$ and $\chi = 0.90$, respectively.

Notice that when trying to get amplification, what matters in the CC model is the entrepreneur's response to the shock in terms of capital accumulation and not his size in the steady state, as measured by his capital holdings. This can be seen when we vary the factor share of capital. The size of the entrepreneur increases relative to the unconstrained model, and so does his capital accumulation response to the shock. However this is not enough to generate amplification to improve upon the model without frictions.

Figure 5: Varying factor shares and χ



The solid and dotted lines depict the CC and RBC models's dynamics with $\alpha = 0.36$ and $\chi = 0.73$, respectively. The dashed and dash-dotted lines depict the CC and RBC models's dynamics with $\alpha = 0.55$ and $\chi = 0.90$, respectively.

Finally, another important reason behind the fact that we are not getting amplification relative to an unconstrained model and relative to what theoretical papers predict is that in our setup, the role of reallocation, which lies at the heart of the amplification mechanism in models with a fixed factor, is not the same. Models with a fixed factor rely on the efficient redistribution from less to more productive agents that is brought about when the price increases after a productivity shock to generate amplification. In our model, where capital is reproducible, this role for reallocation does no longer exist and capital accumulation becomes the key mechanism through which amplification may occur. The intuition behind this is the following. When an economy with reproducible capital is hit by a productivity shock, the initial increase in output is the result of both the shock and the capital accumulation that takes place at the time of the shock. Notice that this holds for both the constrained and unconstrained models. Thus, to get amplification in the model with frictions above that obtained in its frictionless counterpart, capital accumulation in the first should be greater than that in the latter. As stated above, in our model this is not the case, and even though output continues to increase for some periods following the shock, the overall increase is not enough.

3.2.4 Alternative specifications: considering capital adjustment costs and land

Two features that one might think could improve the model's results regarding its lack of amplification are adding capital adjustment costs or land to the benchmark setup. Intuitively, the former would offset, at least partially, the immediate response of investment to a shock which, in turn, would imply higher asset prices relative to the baseline case. On the other hand, the introduction of land as an additional factor of production that also serves as collateral would allow the credit multiplier mechanism to also work through movements in land prices. Despite these seemingly compelling arguments, neither case contributes to getting a significant improvement in the degree of amplification of shocks into the economy in our model. The reason why this is the case has been comented before: RBC-like models mostly rely on capital accumulation (and labor) to produce final output. Furthermore, the fact that there is only one final goods producer using capital as an input implies that the role for reallocation of capital from less to more productive agents, which is key in theoretical papers of fixed assets, is not present here. Note then, that the only way to get amplification in our model is through capital accumulation and not through capital reallocation.[12] Adding adjustment costs inhibits capital accumulation by depressing investment and thus amplification does not occur. With respect to land, even though this provides an additional source through which the credit multiplier can be fed, its relative importance in the productive process is so small that output is hardly affected.[13]

4 The partial equilibrium model

As stated earlier, the most important reason behind the lack of amplification in our benchmark model is the fact that reproducible capital has a strong implication for price volatility. In particular, the general equilibrium dynamics that arise when capital is reproducible dampen the response of its price to a productivity shock, thus preventing the credit multiplier to be triggered. The first three columns of Table 3 illustrate this point by comparing the volatilities of output, investment, price of capital and leverage in the data with the ones obtained from our unconstrained and constrained models.[14]

Table 3

Standard deviations				
	Data	RBC	CC	PE
$\sigma(Y)$	1.60	1.78	1.40	0.92
$\sigma(I)$	7.27	5.67	4.06	4.85
$\sigma(q^K)$	10.35	-	0.56	10.35
$\sigma(lev)$	3.52	-	0.76	14.21

The CC model fares relatively well with respect to output and investment volatility when comparing it to the RBC model, but fails dramatically when it comes to asset prices and leverage. With respect to

[12] In our model, what is "reallocated" when a shock hits the economy, are final goods from households to entrepreneurs. This is, reallocation occurs from consumption to capital production which represents a more productive use of the resources.

[13] The role that land plays as a factor of production for aggregate output is presumably small; hence, the value assigned to the land share in a constant returns to scale aggregate production function should not be too large.

[14] The U.S. time series for output and investment are Real GNP and Gross Private Domestic Investment (in chained 2005 dollars) from the BEA. Capital price is approximated by the S&P 500 market price deflated by the price of non-durable goods and services. Leverage is measured as the liabilities to net worth ratio taken from the Flow of Funds Accounts of the U.S. All variables are detrended using a HP filter from 1952:Q1 to 2009:Q3.

prices, the volatility observed in our model is almost 20 times lower than the one observed in the data, and almost 5 times lower for leverage.

Thus, to prove our claim regarding the implications of reproducible capital, we isolate the credit multiplier mechanism from the general equilibrium dynamics that dampen the response of prices. In order to do this, we develop a partial equilibrium model, where the only two agents are the firm producing consumption goods and the entrepreneur producing capital goods and where the price of capital is modeled as an exogenous stochastic process. As before, we analyze the response of output, investment, price of capital and leverage to a productivity shock in this setup, paying special attention to volatilities, and compare them to those obtained from the data and our original model.

4.1 Firm's problem and financial contract

The consumption goods firm's problem is given as before by:

$$\max_{\{k_t^d,\, l_t^{Ed}\}} F(k_t^d, l_t^{Ed}) - r_t k_t^d - w_t^E l_t^{Ed}$$

where

$$F(k_t^d, l_t^{Ed}) = \theta_t (k_t^d)^{\alpha_k} (l_t^{Ed})^{1-\alpha_k}$$

With respect to the entrepreneur, his problem can now be described as follows. He will begin each period by renting his capital and inelastically supplying his labor to the consumption goods producing firm, and thus his net worth will be:

$$n_t = l_t^E w_t^E + k_t^E r_t$$

The entrepreneur will then engage in a financial contract, using his net worth as the basis of the loan, where he will obtain borrowing from outside the economy, which will again be limited by the value of his collateral. The optimal contract is then given by the solution to:

$$\max_{\{i_t\}} q_t z i_t - d_t$$

$$\begin{aligned} s.t \qquad i_t - n_t &= d_t \\ (1-\chi) q_t z i_t &\geq d_t \\ q_t z i_t - d_t &\geq n_t \end{aligned}$$

He will then invest i_t in his capital goods production technology, and finally, repay the loan. The difference between total capital production in every period and the amount needed to repay the loan will go on to increase the stock of capital in the economy, so that:

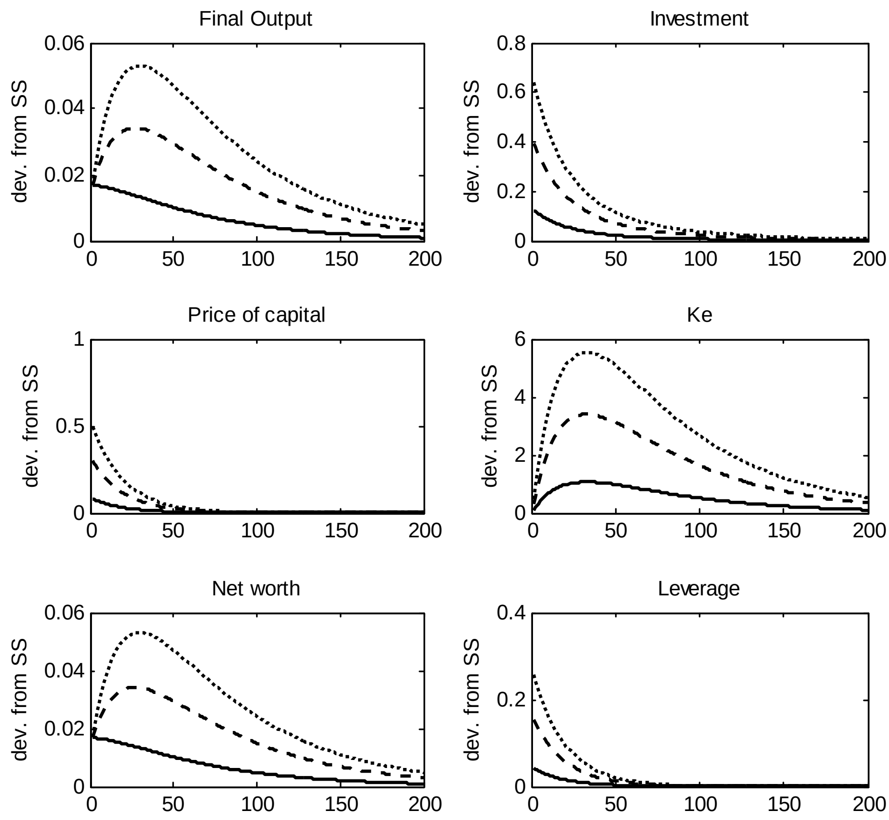
$$k_{t+1} = (1-\delta) k_t + \chi z i_t$$

Finally, the price process is modeled as an AR(1):

$$q_t = a_q + \rho_q q_{t-1} + \varepsilon_t^q$$

with ρ_q and a_q equal to 0.95 and $(1 - \rho_q) * q_{ss}$ respectively, where q_{ss} refers to the steady state value of the price of capital in our original model. To reproduce the fact that the price of capital increases when a productivity shock hits the economy, we assume that these two are correlated, and for simplicity we set the correlation coefficient equal to 1. Finally, we set the standard deviation of the price process σ_q to be equal to 0.081, so as to match the volatility of prices observed in the data. The rest of the parameters take the same values as before. Figure 6 shows the impulse response functions of our partial equilibrium economy when it is hit by a one standard deviation shock to prices, for different values of σ_q.

Figure 6: Impulse response functions



The solid, dashed and dotted lines depict the dynamics of the PE model with σ_q= 0.081, 0.3 and 0.5, respectively.

What we take from this figure is simple: in this setup, the credit multiplier mechanism, when isolated from general equilibrium mechanisms, is effective in amplifying shocks and generating large investment, output, price and leverage fluctuations. The last column of Table 3 presents these variables' volatilities as obtained from the partial equilibrium exercise. As mentioned before, the value of σ_q is set so as to match the volatility of the price of capital so this shouldn't be seen as an accomplishment of the model. What is interesting though, is that at the same time as we match price volatility, our model is able to generate investment volatility of the same magnitude as that obtained from the RBC. This poses an important improvement with respect to the latter, given that even though the RBC model performs well in terms of the real variables, it remains silent when it comes to prices and leverage. With respect to output, we are not able to match its volatility, mainly because we are not allowing for any volatility in labor. Finally, with respect to leverage, the partial equilibrium exercise improves relative to our original model, but overstates volatility in the data quite significantly.

5 Conclusions

In this paper we re-examine the quantitative role of financial frictions, in the form of credit constraints, in business cycles. In particular, we assess whether they are able to generate quantitatively important amplification and propagation effects in a model where capital can be accumulated, relative to a model without frictions. We do this by introducing collateral constraints in the production of capital goods, first in an otherwise standard RBC model, and then in a partial equilibrium setup.

Our results from the general equilibrium model in terms of propagation and amplification do not differ from previous quantitative papers who have embedded the credit multiplier mechanism into RBC models: even though credit constraints delay the velocity at which shocks propagate into the economy, they have no significant amplification effects. However, our main finding is that it is not the credit multiplier what fails in this type of models, but rather their ability to produce sufficient variability in prices. In particular, in a model with reproducible capital like ours, this happens because the general equilibrium effects counteract the logic of price fluctuations described by theoretical models, thus dampening the dynamic interaction between asset prices and credit limits that is necessary for amplification to occur. The partial equilibrium setup allows us to confirm our previous claim: absent the general equilibrium effects, the credit multiplier is indeed an effective amplifying mechanism of shocks into the economy.

This poses a challenge for future research: finding mechanisms that can endogenously generate sufficient variability in asset prices in DSGE models of credit constraints. It is important that these mechanisms do not rely on the type of asset used as collateral or on other special features that may represent important departures from the above mentioned models, which could make the source of amplification, if any, difficult to disentangle.

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