

Expropriation Risk and Aggregate Productivity with Heterogeneous Firms*

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Abstract

In this paper, I propose a general equilibrium model featuring heterogeneous firms and a government that is both unable to commit and relatively more impatient than firms. I find that, as predicted by theoretical papers on limited commitment, the threat of expropriation alone is enough to distort capital accumulation. Moreover, I show that the fact that the government is more impatient than firms induces additional growth dynamics by determining that distortions to capital do not completely go away once the long run stationary equilibrium has been reached. This is because the relative impatience of the government leads not only to decreases in promised utility by the firm when constraints do not bind, but also makes it very costly for a firm to increase its promised utility and capital when a constraint binds. Thus, promised utility will not increase as much as in the case where government and firms discount at the same rate, resulting in a stationary equilibrium level of capital that is less than optimal. Finally, when embedding the contracting problem between a firm and the government in a GE model with heterogeneous firms, I find that expropriation risk is capable of endogenously generating misallocation of resources across firms, with more productive firms being affected the most by the contracting frictions, thus leading to losses in aggregate output and total factor productivity in the long run stationary equilibrium.

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1 Introduction

Resource misallocation can lower aggregate output and total factor productivity. The existing literature on this topic has concentrated mostly on measuring the size of these losses given exogenous distortions to the allocation of resources, and less on finding mechanisms that can endogenously generate these distortions. Motivated by the overwhelming empirical evidence on the existence of expropriation risk, especially in developing countries, in this paper I ask whether this threat is capable of providing such mechanism, that is, can it endogenously generate distortions that lead to resource misallocation and losses in aggregate output and TFP?¹

To answer this question, I first propose a model of firm growth in which there is a government who cannot commit not to expropriate firms and is also more impatient than firms.² I model the interaction between the firm and government as a dynamic contract, and study self-enforcing equilibria in which allocations are constrained by the government's lack of commitment, as in Thomas and Worrall (1994), Albuquerque (2003) and Aguiar and Amador (2009). These papers discuss how the threat of expropriation is enough to distort capital accumulation. A main result of the current paper is that the fact that the government is more impatient than firms induces additional growth dynamics by determining that distortions to capital do not completely go away once the long run stationary equilibrium has been reached. This is because the relative impatience of the government leads not only to decreases in promised utility by the firm when constraints do not bind, but also makes it very costly for a firm to increase its promised utility and capital when a constraint binds. Thus, promised utility will not increase as much as in the case where government and firms discount at the same rate, resulting in a stationary equilibrium level of capital that is less than optimal.³

I then embed this framework into a general equilibrium model with heterogeneous firms, where heterogeneity is modeled as productivity differences. I find that expropriation risk is capable of endogenously generating misallocation of resources across firms, leading to aggregate output and total factor productivity losses in the long run stationary equilibrium. The intuition behind this is the following. Consider a firm that enters a period with a given level of promised utility and must make decisions on investment

¹For evidence on expropriation, see Tomz and Wright (2009).

²There are several political economy reasons that may justify the relative impatience of the government, being one of them the fact that governments may lose office, as in the model of Alesina and Tabellini (1990). In their model, politicians are impatient because the nature of the political process does not assure the current government that it will be reelected. This force for government impatience is present in other political economy models, like Grossman and Van Huyck (1988) and Amador (2004).

³In a forthcoming paper, Acemlogu, Golosov and Tsyvinski study the dynamic taxation of capital and labor in the Ramsey model under the assumption that taxes and public good provision are decided by a self-interested politician who cannot commit to policies. They also allow for the government to be more impatient than firms, and show that in this case the Chamley-Judd result of zero long-run taxes no longer holds, but instead the best subgame perfect equilibrium from the viewpoint of the citizens involves long-run capital taxation.

as well as continuation values for utility that it will give the government contingent on the productivity shock it receives that period. If the firm gets a high shock, one of the things it can do is to choose the optimal level of capital consistent with this shock, but knowing that this would generate incentives for the government to expropriate the firm, it would also have to increase the value of promised utility for the government for next period. Now, given that the government is relatively more impatient than the firm, increasing promised utilities is costly for the firm, and thus it will find it optimal to invest less than the first best amount, reducing the government's incentives to expropriate and the necessary increase in promised utility. If the firm receives instead a low productivity shock, the incentives for the government to expropriate are lower, and thus, so is the cost for the firm, in terms of promised utility, of choosing the optimal level of capital. In this way, the firm will always choose not to distort its optimal decision for capital when it receives a low productivity shock. The relative impatience of the government therefore generates an asymmetry between firms and determines that it will be the more productive firms the ones that are affected the most by the contracting frictions, generating in this way capital misallocation across firms that lead to aggregate output and total factor productivity losses in the long run stationary equilibrium.

In particular, when considering values for the discount factors such that firms are 1.2 times more patient than the government, I get losses in capital, output and total factor productivity that are equal to 36, 26 and 16 percent respectively. These numbers for output and TFP distortions are lower than those documented by Hsieh and Klenow (2009), who use micro data on manufacturing establishments to measure the extent of misallocation in China and India compared to the US. They find sizable gaps in marginal products of labor and capital across plants within narrowly-defined industries and if capital and labor were reallocated to equalize marginal products to the extent observed in the US, manufacturing TFP gains would be of the order of 30 to 50 percent in China and 40 to 60 percent in India. Another related paper is Restuccia and Rogerson (2008), who formulate a version of the growth model where production is carried out by heterogeneous establishments. They calibrate their model to US data and show that policies which create heterogeneity in the prices faced by individual producers can lead to sizeable decreases in output and measured total factor productivity in the range of 30 to 50 percent. It is important to note though, that these numbers arise from exogenous taxes and subsidies to firms that are set to match TFP losses observed in the data. This is different from the current paper in which distortions and misallocation arise endogenously from a contracting problem between the government and the firms, and are therefore more likely to have a hard time in completely explaining observed output and TFP differences. All in all, distortions go in the right direction and explain about half of these differences.⁴

⁴Other papers that look at the role of distortions with firm heterogeneity are Buera and Shin (2008), who study financial frictions, and Khan and Thomas (2009) who look at both financial and real frictions, as possible sources of misallocation

The rest of the paper is organized as follows. Section 2 presents the firm growth model and characterizes the dynamics of an efficient contract. Section 3 describes the general equilibrium framework and aggregate results are shown in Section 4. Finally, Section 5 concludes.

2 The Firm Growth Model

2.1 General Features of the Economy

Time is discrete and runs to infinity. The economy is populated by two agents, a firm and a government. Both have linear utilities and are risk neutral, but the government discounts at a higher rate than firms ($\beta_{gov} = \widehat{\beta} < \beta = \beta_{firm}$). The firm owns the capital and operates technologies that are otherwise unavailable to the government. Their production technology is given by a cobb-douglas decreasing returns to scale function:

$$zk^\alpha l_z^v \tag{1}$$

where z is an idiosyncratic shock to the firm's productivity, which follows a continuous autoregressive process with serial correlation ρ , and unconditional mean \bar{z} :

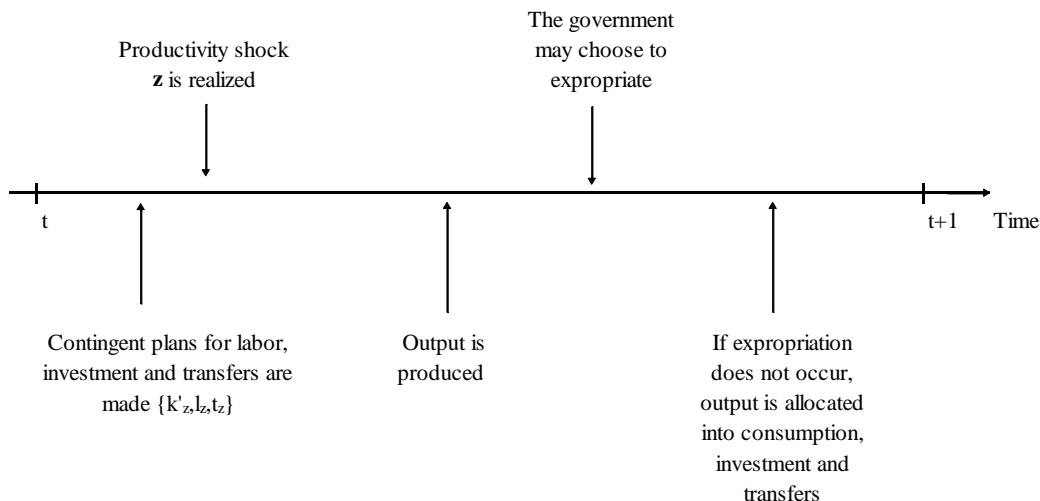
$$\ln z = \rho \ln z_{-1} + (1 - \rho) \ln \bar{z} + \varepsilon \tag{2}$$

with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, k and l_z are the firm's capital and demand for labor respectively, and α and v are their respective shares in output, with $0 < \alpha + v < 1$.

The government cannot commit to not expropriating the firms, and this will give rise to a self-enforcing contract between each firm and the government. Finally, there is no aggregate uncertainty.

Before describing in detail the contracting problem, it will be useful to first present the timeline of the model:

across firms. However, as in Restuccia and Rogerson (2008), the distortions in these papers are also exogenous.



At the beginning of each period, and before the productivity shock is realized, contingent plans for labor, investment and transfers are made. Once the productivity shocks are observed, output is generated. At this stage the government may choose to default on the contract, that is, to expropriate. If expropriation does not occur, output is allocated into consumption, investment and transfers previously determined.

I model the interaction between the firm and government as a contracting problem with limited commitment, and following Spear and Srivastava (1987) and Abreu, Pearce and Stacchetti (1990) approach, I am able to formulate it recursively, as a dynamic program, with a one-dimensional object, promised utility, that summarizes all relevant aspects of agent's history.

2.2 Firms and Government Problem: Self-Enforcing Contract

At the beginning of the period, the long term contract between the government and each firm assigns a utility level T to the government. This lifetime utility level is composed of period transfers t_z (recall that government utility is linear) and a continuation value equal to T_z . Denoting the government's time discount factor by $\widehat{\beta}$, the promise keeping constraint is thus given by:

$$E[t_z + \widehat{\beta}T_z | z_{-1}] = T \quad (3)$$

where $E[./z_{-1}]$ is the conditional expectations operator.

As stated earlier, the government cannot commit to a long term contract and even though firms have commitment, it's of a limited nature in the sense that a participation constraint must be satisfied. In the

same way as Thomas and Worrall (1994) and Albuquerque (2003), I define a self-enforcing contract by requiring that two participation constraints hold. For both types of agents, the participation constraint says that the utility under the contract has to be at least as large as the utility outside the contract. For the government, the participation constraint is:

$$t_z + \widehat{\beta}T_z \geq zk^\alpha l_z^v - wl_z + (1 - \delta)k \quad \forall z \quad (4)$$

where w is the wage. That is, for each possible realization of the productivity shock, lifetime promised utility (period transfers plus discounted value of continuation value) must be at least as big as what the government can obtain if it decided to expropriate, which is given by the firm's output, minus labor payments, plus undepreciated capital⁵. For the firms, the participation constraint limits the long term losses at any time. Denoting the utility function of each firm in state (k'_z, T_z, z) by $U(k'_z, T_z, z)$, where k'_z is capital choice for tomorrow, this restriction states:

$$U(k'_z, T_z, z) \geq 0 \quad \forall z \quad (5)$$

The contract also specifies how each firm allocates output to new investment, transfers to the government and payments to households, in the form of labor income and profits:

$$zk^\alpha l_z^v = k'_z - (1 - \delta)k + t_z + wl_z + \pi_z \quad (6)$$

where $k'_z - (1 - \delta)k$ is investment, and π_z are the period profits. We can obtain an expression for firm profits by rearranging the previous expression:

$$\pi_z = zk^\alpha l_z^v - k'_z + (1 - \delta)k - wl_z - t_z \quad (7)$$

Finally, with respect to transfers, I will assume that they cannot be negative and cannot exceed output net of investment and wage payments (i.e. profits as defined above cannot be negative). Thus, a contract is feasible if:

$$t_z \geq 0 \quad \forall z \quad (8)$$

$$t_z \leq zk^\alpha l_z^v - k'_z + (1 - \delta)k - wl_z \quad \forall z \quad (9)$$

Summing up, each firm chooses contingent plans for labor (l_z), capital (k'_z), transfers (t_z) and promised utility (T_z) to maximize lifetime expected profits. The firm's problem is thus:

$$\max_{\{k'_z, l_z, t_z, T_z\}} E[\pi_z + \beta U(k'_z, T_z, z) / z_{-1}]$$

subject to the promise keeping, participation and feasibility constraints.

⁵Recall that the firm's technology cannot be operated by the government.

2.3 Dynamics of an Efficient Contract

Let $\lambda_1, \lambda_2(z)p(z/z_{-1}), \beta\lambda_3(z)p(z/z_{-1}), \lambda_4(z)p(z/z_{-1})$ and $\lambda_5(z)p(z/z_{-1})$ be the multipliers for the promise keeping constraint, both participation constraints and both feasibility constraints respectively. The first order conditions for the firm's problem are:

$$\begin{aligned} (zk^\alpha vl_z^{v-1} - w)(1 - \lambda_2(z) + \lambda_5(z)) &= 0 \\ \beta E [(1 - \lambda_2(z') + \lambda_5(z')) (z' \alpha k_z'^{\alpha-1} l_z^v + (1 - \delta)) / z] &= 1 + \lambda_5(z) \\ \lambda_1 + \lambda_2(z) + \lambda_4(z) - \lambda_5(z) &= 1 \\ \widehat{\beta}(\lambda_1 + \lambda_2(z)) &= -\beta(1 + \lambda_3(z))U'(k, T_z, z) \end{aligned}$$

for labor, capital, transfers and promised utility respectively. The envelope condition is:

$$U'(k, T, z_{-1}) = -\lambda_1$$

which together with the first order condition for promised utility gives:

$$U'_{T_z}(k'_z, T_z, z) = \frac{\widehat{\beta}}{\beta}((1 + \lambda_3(z))U'_{T_z}(k, T, z) - \lambda_2(z))$$

The first condition shows that labor will not be affected by the constraints, given that equating the wage with the marginal product of labor is always possible, and thus labor choice will always be optimal. Intuitively, it is easy to see why this would be the case, as the government is threatening firm's output after paying labor income and this provides incentives for the firm to want to hire labor up to the optimum level as this will lower profits and thus the probability of being expropriated.

The second condition determines the optimal level of investment by each firm which will also depend on both the participation constraints for the government and the feasibility constraints for profits. The key thing to note here is that capital will be less than its efficient level as long as at least one of the participation constraints is binding. The third condition tells us that the value of transfers will depend on the envelope condition, the participation constraint of the government and the feasibility constraints. Finally, the last condition describes the trade-offs across different states of nature when choosing continuation utility levels, in terms of firm value. This ultimately depends on the level of promised utility, the commitment constraints and the relative impatience of the government.

It will be helpful to start the analysis by considering the first-best situation where the self-enforcing constraints are ignored, that is, when there is full commitment by both agents. This will serve as a benchmark for comparison and will help understand the role of commitment in generating distortions both in capital allocation and productivity.

2.3.1 The Perfect Enforcement Solution

Eliminating the participation or self-enforcing constraints from the problem, that is, setting $\lambda_2(z) = \lambda_3(z) = 0$, yields the following solution. Optimal labor demand is given by:

$$l_z^* = \left(\frac{vzk^\alpha}{w} \right)^{\frac{1}{1-\nu}}$$

and capital is such that its marginal product is equal to the interest rate:⁶

$$\begin{aligned} f'(k_z^*) &= \frac{1}{\beta} - 1 + \delta \\ k_z^* &= E \left[(z'^{\frac{1}{1-\nu}}) / z \right]^{\frac{1-\nu}{1-\alpha-\nu}} \left(\frac{\alpha\beta \left(\frac{v}{w} \right)^{\frac{v}{1-\nu}}}{1-\beta} \right)^{\frac{1-\nu}{1-\alpha-\nu}} \end{aligned}$$

Clearly the efficient first best contract which maximizes the firm's payoff involves an investment level k_z^* each period and no transfers, so the government gets its reservation payoff of zero. For any other point on the Pareto frontier, given by $U^*(k, T, z_{-1}) = \frac{\pi^*}{(1-\beta)} - T$, where π^* represent the optimal per-period profits, the corresponding contract still involves investment k_z^* each period but positive transfers which must satisfy the feasibility constraints and whose time path will be governed by the difference in discounting between the firms and government. It is important to note that there will be no transition dynamics in this perfect enforcement model, as there are no restrictions preventing the firms to move to their first best allocations instantaneously.

2.3.2 The Constrained Solution

When including the self-enforcing constraints, quite a few things change relative to the perfect enforcement solution. First, I find that, as predicted by theoretical papers on limited commitment, the threat of expropriation alone is enough to distort equilibrium factor allocations, and slow capital accumulation. However, in my model investment does not evolve "ratchet-like" as these papers predict, that is, it may decrease over time, and this is mainly due to the fact that the government discounts at a higher rate than firms. In particular, even though investment does move almost one-to-one with promised utility, the latter will not be monotonic, as it may increase when at least one of the participation constraints is binding, but will decrease whenever they are not. These induces additional growth dynamics, relative to the ones observed in a model with limited commitment in which both agents discount at the same rate. In particular, the relative impatience of the government makes it very costly for a firm to increase its promised utility and capital when a constraint binds, and thus promised utility will not increase as much, resulting in capital distortions do not completely go away once the long run equilibrium is reached.

I will begin by showing analytically how these dynamics may arise and then go on to illustrate them by simulating a firm's growth path. The key to understanding and solving the optimal contract is to

⁶I am assuming here that feasibility constraints do not bind in the perfect enforcement solution.

note that T , the promised utility to the government, is the state variable of the dynamic programming problem that will ultimately determine, combined with the first order conditions, the values of investment, transfers and continuation utility. Once the new state is known, this, together with the chosen values of T_z and k'_z , will determine the next period's state and the process is repeated. Let's assume that a firm begins the period with promised utility T , productivity z_{-1} and capital level k and first consider how T_z is related to T . Recall the first order and envelope conditions for promised utility:

$$U'(k'_z, T_z, z) = \frac{\widehat{\beta}}{\beta} ((1 + \lambda_3(z))U'(k, T_z, z) - \lambda_2(z))$$

From here, we can see that given that $\lambda_2(z) \geq 0$ and $\lambda_3(z) \geq 0$, the following is true:

$$U'(k'_z, T_z, z) \leq \frac{\widehat{\beta}}{\beta} U'(k, T_z, z)$$

Proposition 1: If a state occurs where the self-enforcing constraint does not bind, $\lambda_2(z) = 0$ and $U'(k, T_z, z) = \frac{\widehat{\beta}}{\beta} U'(k_{-1}, T, z_{-1})$. Given that $U'(k_{-1}, T, z_{-1}) < 0$ and $\widehat{\beta} < \beta$, then $U'(k'_z, T_z, z) > U'(k, T, z_{-1})$ and $T_z < T$. That is, promised utility will decrease whenever the participation constraint does not bind. If on the other hand a state occurs in which it does bind, $\lambda_2(z) > 0$ and $U'(k, T_z, z) < \frac{\widehat{\beta}}{\beta} U'(k_{-1}, T, z_{-1})$. In this case, $U'(k'_z, T_z, z)$ can be greater, equal or smaller than $U'(k, T, z_{-1})$ and thus T_z can decrease, stay the same or increase relative to T . This will depend on the value of the discount factors, the slope of the firm's value function and the value of the relevant multiplier.

As stated before, the relative impatience of the government is key here, as it will make it very costly in terms of continuation utility, for a firm who receives a high shock to increase its capital to the optimal level consistent with this shock, if it doesn't want to be expropriated. Thus, it will choose to operate at a lower than optimal level of capital, reducing the incentives for the government to expropriate and the necessary continuation values. Having said this, two caveats regarding the behavior of continuation utility T_z are in place, and these are summarized in the following corollary:

Corollary 2: For low levels of promised utility T , where the feasibility constraint stating transfers cannot be negative is most likely to bind, continuation utility T_z will be lower than otherwise, to ensure that the constraint is satisfied. On the other hand, for high levels of T , where the feasibility constraint stating profits cannot be negative is more likely to bind, continuation utility T_z will be higher than otherwise, to ensure that the constraint is satisfied.

With respect to capital, its dynamics are described in the second proposition of the paper, and result directly from its first order condition:

Proposition 3: If participation constraints do not bind, investment will be equal to its optimal level, but if at least one participation constraint binds, then capital will be lower than its efficient level. In these cases, investment evolves one-to-one with continuation utility. That is, if a state occurs such that there is a large temptation from the government to renege on the contract, this will lead to a higher value of continuation utility and hence a higher value of investment.

Finally, transfers will be determined by the promise keeping, participation and the feasibility constraints. Proposition 4 directly follows from corollary 2:

Proposition 4: Transfers will be equal to zero for low values of promised utility, weakly increase for intermediate values and then be such that profits are equal to zero for the highest values of promised utility.

Note that this is different from the case where $\hat{\beta} = \beta$, in which transfers are zero for all periods until the period before the maximum value of promised utility is reached.

Figure 1 illustrates the above results. In particular, Figure 1a. shows how continuation values of promised utility behave as the participation constraints bind or not. We can clearly see how every time the constraint binds, shown by the shaded areas, promised utility stays the same or increases, while it decreases when the constraint does not bind (unshaded areas). An exception to this pattern can be seen between periods 30 and 40, where promised utility increases, even though participation constraints are not binding. As stated in corollary 2, this can be explained by the fact that feasibility constraints are binding, and thus promised utility must increase to ensure the constraint is satisfied. Figure 1b. shows the close to one-to-one relationship between promised utility and capital, where the solid line represents promised utility and the dotted one represents capital. Given that capital is also a function of the productivity shocks, capital may fluctuate even if promised utility does not change, reflecting differences in productivity.

Figure 1a.

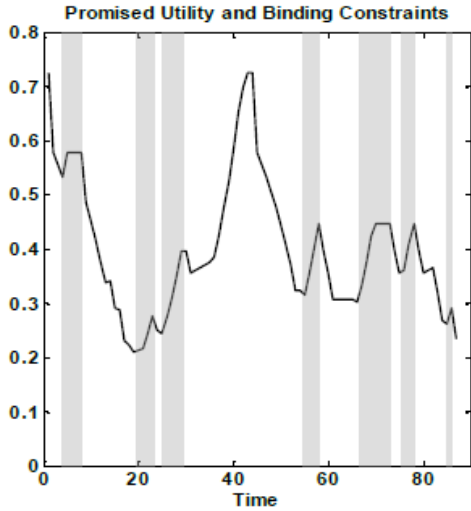
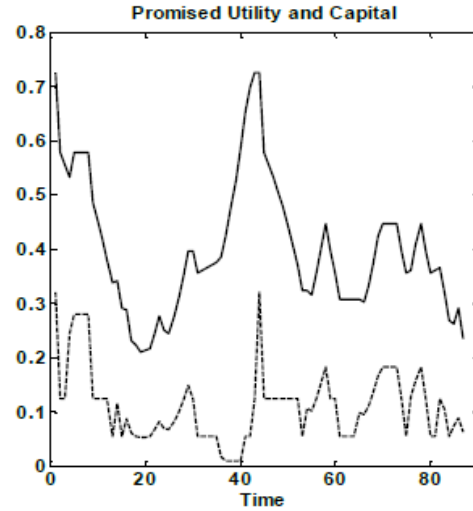


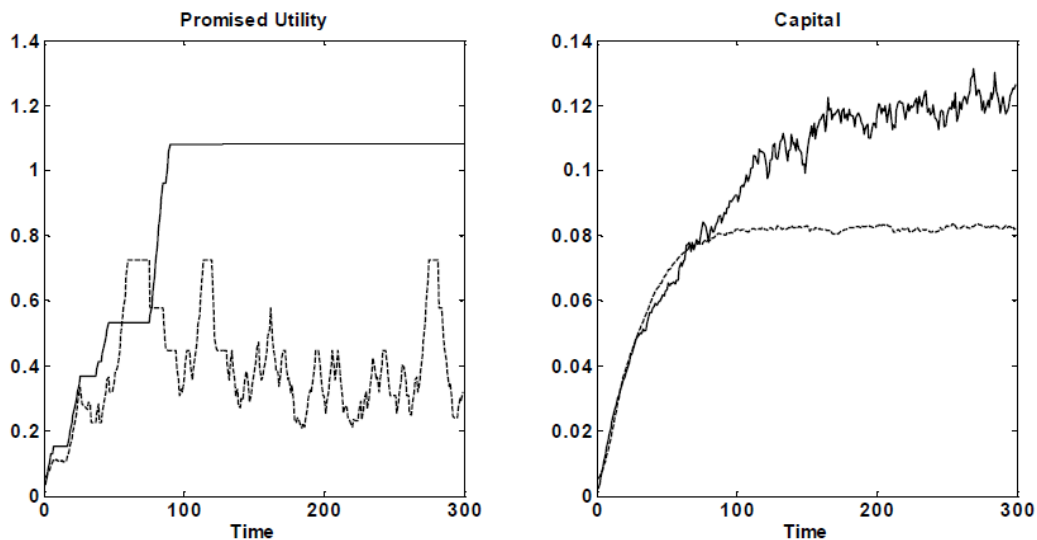
Figure 1b.



Finally, Figure 2 helps illustrate the key role of impatience in the model by comparing the behavior of promised utility and capital with those of a model where firms and government discount at the same

rate.

Figure 2



This figure presents the time 0 unconditional expected path dynamics implied by the contract. The dotted lines depict variables in the current model, that is, with relative impatience of the government, while the solid lines illustrate their counterparts in a model where discount factors are the same. With respect to promised utility, it can be seen how it behaves quite differently in the two models. When discount factors are the same, promised utility is non-decreasing, remaining constant whenever the participation constraints do not bind and increasing when they do. This is different from this model in which promised utility falls whenever these constraints do not bind. Not only this, but the level of promised utility in the stationary equilibrium is lower than in the model with equal discount factors. The same can be said with respect to capital: while in the latter case capital increases until it reaches its optimal level, in the presence of a relatively more impatient government capital distortions do not completely go away in the long run stationary equilibrium, and thus, the first best levels are not attained.

3 General Equilibrium

Consider now an economy populated by three types of agents, a large number of firms with heterogeneous productivity, a government, and households, all of which are infinitely lived. Households are identical. I will now go on to present the household's problem, then define a recursive competitive equilibrium and finally present the quantitative results.

3.1 Household's Problem

The representative household's problem is given by:

$$U^H(b) = \max_{c, l^h, b'} [u(c, 1 - l^h) + \beta U^H(b')]$$

s.t.

$$c + b' \leq w l^h + r b + \pi$$

where $\beta \in (0, 1)$ is the time discount factor, c , b , and l^H are household's consumption, stock of bonds and labor supply, respectively. Leisure endowment is normalized to one. Every period, the household sells her labor at a wage rate w and receives income from last period's bonds at the rate r as well as undistributed profits from the firms⁷. They also purchase consumption goods and new bonds, both at a price of 1.

Household's choices are summarized in the Euler equation:

$$u_1(c, 1 - l^h) = \beta r u_1(c', 1 - l'^h)$$

and labor is supplied inelastically:

$$l^h = \bar{l}$$

3.2 Equilibrium

A recursive competitive equilibrium is a set of functions $c(b; \mu)$, l^h and $b'(b; \mu)$ for the households and $U(k, T, z_{-1}; \mu)$, $\{k'_z(k, T, z_{-1}; \mu)$, $l_z(k, T, z_{-1}; \mu)$, $t_z(k, T, z_{-1}; \mu)$ and $T_z(k, T, z_{-1}; \mu)\}_{z=1}^{Nz}$ for the firms such that:

1. Given prices $w(\mu)$ and $r(\mu)$, policy functions solve the household and firms problems
2. Markets for bonds, labor and output clear:

$$\sum b'(b; \mu) = 0$$

$$l^h = \sum_{S=k*T*z_{-1}} l_z(k, T, z_{-1}; \mu) \mu(S) = \frac{1}{3}$$

$$\sum_S z k^\alpha l_z^v(k, T, z_{-1}; \mu) \mu(S) = c(b; \mu) + \sum_S [k'_z(k, T, z_{-1}; \mu)$$

$$-(1 - \delta)k + t_z(k, T, z_{-1}; \mu)] \mu(S)$$

where S is the product space $k * T * z_{-1}$ and μ is the distribution of firms over k, T and z_{-1} .

⁷I assume here that households own the firms.

3. The joint distribution of capital, productivity and promised utilities evolves according to the following mapping:

$$\mu'(k'_z, T_z, z) = \sum_{k, T, z_{-1}/k'_z(), T_z() \in (k'_z, T_z)} p(z/z_{-1}) \mu(S)$$

where $p(z/z_{-1})$ is the conditional density of the productivity shock.

4 Quantitative Properties of the Model

In this section I present the results from the numerical simulations of the model. I start by analyzing the path dynamics of the model, paying special attention to the long run stationary state, and comparing how results change when varying the degree of relative impatience of the government. I then extend the model to allow for exogenous contract terminations and show how results change in this setting.

4.1 Calibration

Table 1 summarizes the baseline parameter choices:

Firms and HHs discount factor	β	0.96
Gov's discount factor	$\widehat{\beta}$	0.8 / 0.88
Capital share	α	0.3
Labor share	ν	0.55
Depreciation	δ	1
Autoregressive coeff. productivity shock	ρ	0.95
Unconditional mean of shock	\bar{z}	1
Std. dev. productivity shock	σ_z	0.25

The period considered is one year. The discount factor for firms and households is borrowed from the real business cycles literature, while the choice of government's discount factor is less clear. The literature that features relative impatience of the government is mostly theoretical, and there is no unique stand on the value this parameter should take. Hence, I decided to work with different degrees of impatience, in particular, 0.8 and 0.88, and compare the results. Capital and labor shares are set to match estimates of establishment-level production functions.⁸ For tractability purposes only, I assume full depreciation.

To solve the model I discretize the state space. The idiosyncratic productivity shock can take on one of 5 possible values, with unconditional mean equal to one. The autocorrelation coefficient is equal to 0.95 and the volatility of the shock is set to match firm volatilities documented by Davis et al (2006).

⁸See Restuccia and Rogerson (2008).

The transition matrix $[\pi_{ij}]$, with $\pi_{ij} = \Pr[z = z_j | z_{-1} = z_i]$, is chosen to be a discrete state space representation of the autoregressive process (2). This is done with the numerical quadrature method developed by Tauchen and Hussey (1991). To calibrate the distribution of the initial shock, $F(z_{-1})$, I use the invariant distribution induced by the transition matrix $[\pi_{ij}]$.

For the values of promised lifetime utility for the government, I choose a logspaced grid of 200 points starting in \underline{T} and ending in \bar{T} . I pick a sufficiently high upper bound for T such that at this level, the firms are unconstrained for all shocks. Also, this value does not restrict the optimal solution since $U(\bar{T}, z_j) < 0$ for all j . With respect to \underline{T} , I choose it to be 1 percent below the autarky level of lifetime utility which is capable of sustaining the optimal unconstrained choice of capital for the lowest realization of the shock. Initial investment is such that all firms start with promised utility equal to \underline{T} .

4.2 Results

I solve for the stationary equilibrium of this economy based on the fixed-point algorithm of Aiyagari (1994), iterating on the wage until labor market clears in the stationary equilibrium. For a given guess on the wage, I solve the firms' problem, and given the optimal decision rules, I simulate N firms for S periods. I set $N = 20,000$ and S , which is the period by which the economy must have reached the steady state, to 300 years. Finally, I verify that increasing S has virtually no effect on the invariant distribution. Table 2 summarizes the results of this exercise:

Table 2

	$\hat{\beta} = 0.8$	$\hat{\beta} = 0.88$	$\hat{\beta} = \beta$
$(k/k^*)_{ss}$	0.64	0.70	1
$(y/y^*)_{ss}$	0.74	0.78	1
$(tfp/TFP^*)_{ss}$	0.84	0.87	1

This table shows the values of capital, output and total factor productivity in the model with limited commitment relative to those in the perfect enforcement model. One of the things that stands out is that when the government discounts at a higher rate, distortions to capital and output don't go away once the long run stationary equilibrium has been reached. Even more, in this model we get distortions to total factor productivity in the stationary state, and the resulting losses become more important the more impatient the government is. In particular, when the government's discount factor is equal to 0.8, losses in capital, output and TFP are 36, 26 and 16 percent respectively. This is not the case for a model where government and firms discount at the same rate, in which capital, output and TFP achieve their first best levels in the steady state. Thus, we can conclude that expropriation risk, together with a relatively more impatient government, are capable of endogenously generating misallocation of resources

across firms, leading to aggregate output and total factor productivity losses in the long run stationary equilibrium.

The intuition behind this is the following. Take a firm that enters a period with a given level of promised utility and must make decisions on investment as well as continuation values for utility that it will give the government contingent on the productivity shock it receives that period. If it gets a high shock, one of the things it can do is to choose the optimal level of capital consistent with this shock, but knowing that this would generate incentives for the government to expropriate the firm, it would also have to increase the value of promised utility for the government for next period. Now, given that the government is relatively more impatient than the firm, increasing promised utilities is costly for the firm, and thus it will prefer to invest less than the optimal amount, reducing the government's incentives to expropriate and the necessary increase in promised utility. If the firm receives instead a low productivity shock, the incentives for the government to expropriate are lower, and thus, so is the cost for the firm, in terms of promised utility, of choosing the optimal level of capital. In this way, the firm will always choose not to distort its optimal decision for capital when it receives a low productivity shock. The relative impatience of the government therefore determines that it will be the more productive firms the ones that are affected the most by the contracting frictions, generating in this way capital misallocation across firms that lead to aggregate output and total factor productivity losses in the long run stationary equilibrium.

4.3 Model with Exogenous Entry and Exit

Let $\gamma > 0$ be the exogenous exit probability. Exit is assumed to occur at the end of the period after production has taken place and transfers are paid out. If a firm exits the contract, a new contract is offered starting at a level of promised utility that is consistent with the average size of entering firms observed in the data (T_{\min}).

The problem of the firm is now the following:

$$U(k, T, z_{-1}) = \max_{\{k'_z, l_z, t_z, T_z\}} E \left\{ \begin{array}{l} zk^\alpha l_z^v - k'_z + (1 - \delta)k \\ -wl_z - t_z \\ +\beta(1 - \gamma)U(k'_z, T_z, z) \end{array} \middle| z_{-1} \right\}$$

s.t.

$$\begin{aligned}
E[t_z + \widehat{\beta}(1 - \gamma)T_z + \widehat{\beta}\gamma T_{\min}|z_{-1}] &= T \\
t_z + \widehat{\beta}T_z &\geq zk^\alpha l_z^v - wl_z + (1 - \delta)k \quad \forall z \\
U(k'_z, T_z, z) &\geq 0 \quad \forall z \\
t_z &\geq 0 \quad \forall z \\
zk^\alpha l_z^v - k'_z + (1 - \delta)k - wl_z &\geq t_z \quad \forall z
\end{aligned}$$

To solve this model, I use the baseline parametrization and set $\gamma = 0.05$, which is lower than what most papers with exogenous death probability use. It will now become clear why I choose this lower value. Table 3 summarizes the results from this exercise, and compares them to our previous results:

Table 3

		$\widehat{\beta}=0.8$	$\widehat{\beta}=0.88$	$\widehat{\beta}=\beta$
$\gamma = 0$	$(k/k^*)_{ss}$	0.64	0.70	1
	$(y/y^*)_{ss}$	0.74	0.78	1
	$(tfp/TFP^*)_{ss}$	0.84	0.87	1
$\gamma = 0.05$	$(k/k^*)_{ss}$	0.68	0.73	0.95
	$(y/y^*)_{ss}$	0.76	0.80	0.97
	$(tfp/TFP^*)_{ss}$	0.85	0.89	0.98

Two things stand out. First, when considering a model where firms and government discount at the same rate (third column), having exogenous death leads to some firms being distorted in the stationary equilibrium, even though the resulting losses are quite small. However, in the presence of a relatively more impatient government, distortions are bigger when the probability of exogenous exit is zero. The reason for this is that having a positive exogenous death probability introduces two counteracting mechanisms in the model. On the one hand, there are more firms with low levels of promised utility, and thus, low levels of capital, but on the other hand, the difference between the discount factors of the firms and government is reduced, hence lowering the relative impatience effect. Given that the latter is the strongest in generating potential losses in output and TFP in the steady state, reducing this effect leads to lower overall distortions, relative to a model where exogenous death is not present.

5 Conclusions

In this paper I present a model of firm growth where the government cannot commit not to expropriate firms and is more impatient than firms. This model displays capital distortions that do not go away in

the long run stationary equilibrium, and faster speed of convergence relative to a model with limited commitment but the same degree of impatience. This is because the relative impatience of the government leads not only to decreases in promised utility by the firm when constraints do not bind, but also makes it very costly for a firm to increase its promised utility and capital when a constraint binds. Thus, promised utility will not increase as much as in the case where government and firms discount at the same rate, resulting in a stationary equilibrium level of capital that is less than optimal.

Embedding this model in a general equilibrium framework with heterogeneous firms, I find that expropriation risk is capable of endogenously generating misallocation of resources across firms, with more productive firms being affected the most by the contracting frictions, thus leading to losses in aggregate output and total factor productivity in the long run stationary equilibrium.

6 References

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